Double-Tee worked example

Prestressed Double-Tee Slabs Structural Design Report

Notation

$\%_{ps1}$	=	Percentage of reinforcement required to resist M_{u1}
$\%_{ps2}$	=	Percentage of reinforcement left to resist M_{u2}
$\%_{ps}$	=	Ratio of initial jacking stress to ultimate prestressing steel strength(f_{psi}/f_{psu})
A'_s	=	Sum of cross-sectional areas of all mild longitudinal compression reinforcement
A_c	=	Cross-sectional area of composite section
A_p	=	Cross-sectional area of precast section
A_{ps}	=	Sum of cross-sectional areas of all strands
$A_{s,ft,req}$	=	Total amount of bonded rebar required to carry the total tensile load if tensile stress limits at transfer stage are exceeded
A_s	=	Sum of cross-sectional areas of all mild longitudinal tension reinforcement
E_p	=	Elasticity Modulus of precast concrete
E_{pi}	=	Elasticity Modulus of precast concrete at transfer stage
E_{ps}	=	Elasticity Modulus of prestressing strands
E_t	=	Elasticity Modulus of topping concrete
I_c	=	Gross moment of inertia of composite section
I_{cr}	=	Moment of inertia of cracked composite section
I_e	=	Effective moment of inertia of composite section
I_p	=	Gross moment of inertia of precast section
M_{LL}	=	Bending moment due to live loads
M_{SDL}	=	Bending moment due to super imposed dead loads
M_{SW}	=	Bending moment due to self weight
M_a	=	Maximum moment due to service loads at the stage deflections are calculated
M_{cr}	=	Cracking moment of the section
M_{cre}	=	Moment causing flexural cracking at section due to externally applied loads
M_{max}	=	Maximum factored moment at section due to externally applied loads
M_{r1}	=	Ultimate moment capacity of section at Stage 1
M_{r2}	=	Ultimate moment capacity of section at Stage 2
M_{to}	=	Bending moment due to topping weight
M_{u1}	=	Design moment at Stage 1 for bending moment strength analysis
M_{u2}	=	(Additional) design moment at Stage 2 for bending moment strength analysis
RH	=	Relative humidity
S_{bc}	=	Section modulus of bottom fiber of composite section
S_{bp}	=	Section modulus of bottom fiber of precast section

S_{tc} = Section modulus of top fiber of composite section

S_{tp} = Section modulus of top fiber of precast section

V/S = Volume to surface ratio of the member

V_c = Nominal shear strength provided by concrete

V_c = Nominal shear strength provided by shear reinforcement

V_{ci} = Flexure-shear strength of concrete section

V_{cw} = Web-shear strength of concrete section

V_d = Total shear force due to unfactored dead load

V_i = Total shear force due to unfactored loads other than dead load

V_{ps} = Maximum factored moment at section due to externally applied loads

V_u = Factored shear force at section

d_{ps} = Distance from extreme compressive fiber to centroid of prestressing reinforcement

d_s = Distance from extreme compressive fiber to centroid of mild rebar tension reinforcement

f'_{cp} = Compressive strength of precast concrete

f'_{cpi} = Compressive strength of precast concrete at transfer stage

f'_{ct} = Compressive strength of topping concrete

f'_{psi} = Stress at prestressing strand at the stage for which stress loss is calculated

$f_{cp1,bot}$ = The stress at extreme bottom fiber of precast concrete section at Stage 1

$f_{cp1,max}$ = The maximum (tensile) stress in precast concrete section at Stage 1

$f_{cp1,min}$ = The minimum (compressive) stress in precast concrete section at Stage 1

$f_{cp1,top}$ = The stress at extreme top fiber of precast concrete section at Stage 1

$f_{cp2,bot}$ = The stress at extreme bottom fiber of precast concrete section at Stage 2

$f_{cp2,max}$ = The maximum (tensile) stress in precast concrete section at Stage 2

$f_{cp2,min}$ = The minimum (compressive) stress in precast concrete section at Stage 2

$f_{cp2,top}$ = The stress at extreme top fiber of precast concrete section at Stage 2

$f_{cp3,bot}$ = The stress at extreme bottom fiber of precast concrete section at Stage 3

$f_{cp3,max}$ = The maximum (tensile) stress in precast concrete section at Stage 3

$f_{cp3,min}$ = The minimum (compressive) stress in precast concrete section at Stage 3

$f_{cp3,top}$ = The stress at extreme top fiber of precast concrete section at Stage 3

$f_{cps,lt}$ = Stress at prestressing strand level due to sustained loads

f_{cps} = Stress at prestressing strand level

$f_{ct3,top}$ = The stress at extreme top fiber of topping layer at Stage 3

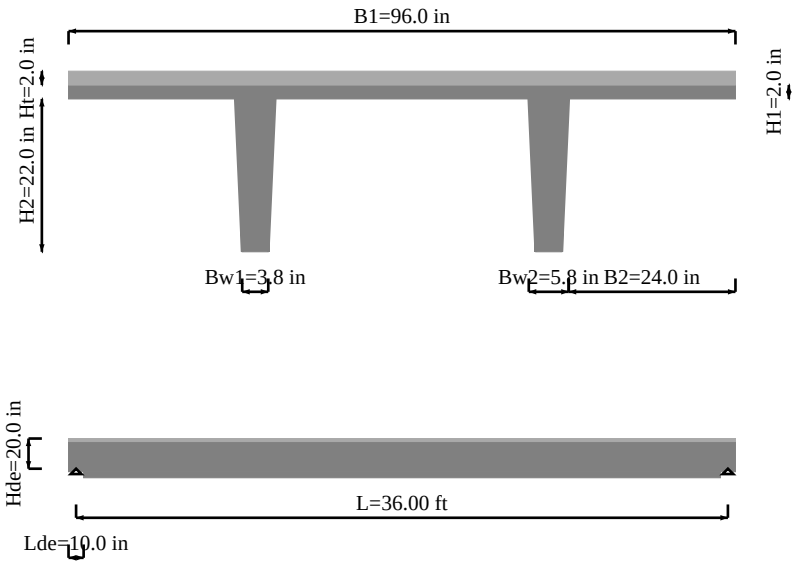
f_{pe} = Compressive stress in concrete due only to effective prestressing forces at extreme fiber of section

f_{ps} = Stress at prestressing strand for nominal moment strength calculation

f_{psc}	=	Compressive stress in concrete, after allowance for all prestress losses, at centroid of cross section resisting externally applied loads or at junction of web and flange where the centroid lies within the flange
f_{psu}	=	Rupture stress of prestressing strand
f_{psy}	=	Yield stress of prestressing strand
f_{rp}	=	Modulus of rupture for precast concrete
f_{sy}	=	Yield stress of mild rebar tension reinforcement
n	=	Modular ratio of reinforcing (E_{ps}/E_p)
t_1	=	Time that passed between jacking of strands and the time for which relaxation loss at transfer stage is calculated
t_2	=	Time that passed between jacking of strands and the time for which relaxation loss at service stage is calculated
xL	=	Distance of the section from left end of the member
y_{bc}	=	Distance from neutral axis to the bottom fiber of composite section
y_{bp}	=	Distance from neutral axis to the bottom fiber of precast section
y_{psc}	=	Distance from neutral axis of composite section to the centroid of prestressing strands
y_{psp}	=	Distance from neutral axis of precast section to the centroid of prestressing strands
y_{tc}	=	Distance from neutral axis to the top fiber of composite section
y_{tp}	=	Distance from neutral axis to the top fiber of precast section
$\Delta f_{ps,cr}$	=	Loss at prestressing stress due to creep
$\Delta f_{ps,el}$	=	Loss at prestressing stress due to elastic shortening of the member
$\Delta f_{ps,re1}$	=	Loss at prestressing stress due to relaxation of strands at transfer stage
$\Delta f_{ps,re2}$	=	Loss at prestressing stress due to relaxation of strands at service stage
$\Delta f_{ps,sh}$	=	Loss at prestressing stress due to shrinkage
$\Sigma P(t)$	=	Total force in prestressing strands at time t for which stress is calculated
ΣP_e	=	Total force of all prestressing strands after all losses
ΣP_i	=	Initial jacking force for all prestressing strands
β_1	=	Factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis
γ_{ps}	=	Factor used for type of prestressing reinforcement for nominal moment strength calculation
ρ'_s	=	Ratio of A'_s to bd_s
ρ_{ps}	=	Ratio of A_{ps} to bd_{ps}
ρ_s	=	Ratio of A_s to bd_s
ϕ	=	Strength reduction factor
ϕ_{ps}	=	Diameter of prestressing strands
$\phi_{vs,prov}$	=	Shear reinforcement provided (in one of the beams) at relevant region

1. Analysis and Design Parameters

1.1. Section and member dimensions



1.2. Material

Precast concrete, f_{cp}	=	5000 psi
Topping concrete, f_{ct}	=	3000 psi
Prestressing strands, f_{psu}	=	270000 psi
Prestressing strands, f_{psy}	=	229500 psi
ϕ_{ps}	=	0.5"
$\%_{ps}$	=	75 %
Rebars, f_{sy}	=	60000 psi

1.3. Loads

Superimposed dead load	=	15.00 lb/ft^2
Live load	=	66.00 lb/ft^2

1.4. Prestressing loss input

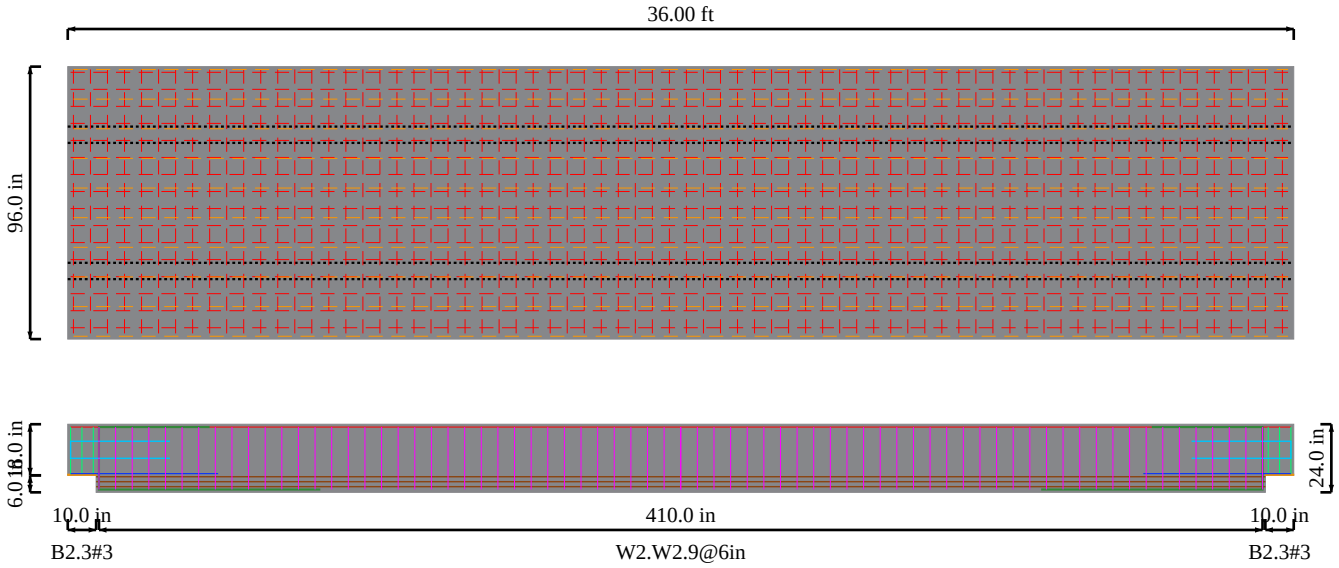
Relative humidity, RH	=	70 %
t_1	=	24 hours
t_2	=	87600 hours (10 years)

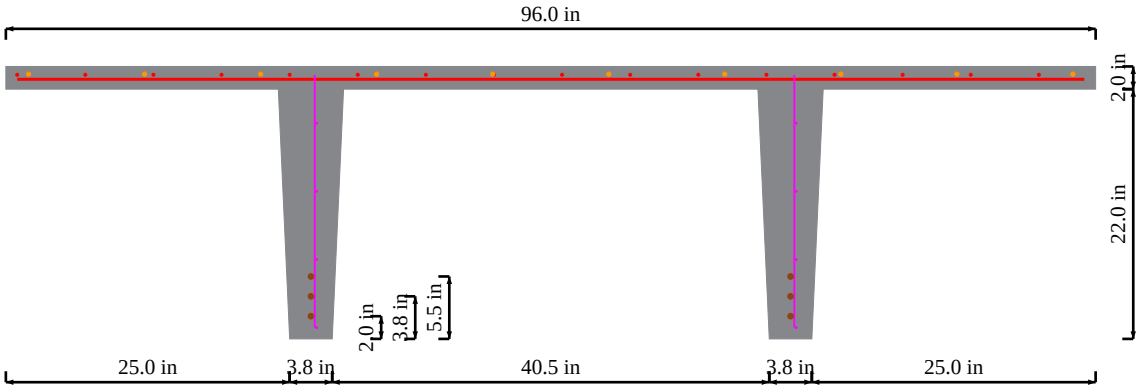
Composite action is taken into account in analysis and design. The member is designed fully prestressed. In other words, bending moment strength is provided only by prestressing strands.

2. Design Results

f_{cpi} = 3875 psi ($0.78f_{cp}$)

2.1 Design Drawings





2.2 Quantity Takeoff

Label	Color	Size	Qty	L(ft)	A(ft^2)	V(ft^3)	W(lbs)
Concrete							
5000 psi		-	-	-	-	99.71	-
PS Strands							
PS1 (270K)		0.50"	6	34.33	-	-	**107.04**
Mild Steel Rebars							
B1		#3	10	35.83	-	-	134.33
B2		#3	6	1.38	-	-	3.11
B3		#6	2	11.67	-	-	34.99
B4		#4	4	4.33	-	-	11.55
B5		#4	4	6.33	-	-	16.88
Total							**200.86**
Wire Mesh							
W1		W5.5@6in	-	-	280.69	-	209.71
W2		W2.9@6in	-	-	62.64	-	24.68
Total							**234.39**
Steel Plates							
t=0.40in		10inx4in	2	-	-	-	9.11

Table 2.2.1 Quantity takeoff list

3. Preliminary Calculations

	Precast Section	Composite Section
Cross-sectional area of the section, A	401 in^2	593 in^2
Distance from neutral axis to the bottom fiber, y_b	17.1 in	19.3 in
Distance from neutral axis to the top fiber, y_t	6.9 in	6.7 in
Gross moment of inertia of the section, I	20986 in^4	27724 in^4
Section modulus of bottom fiber, S_b	1227 in^3	1436 in^3
Section modulus of top fiber, S_t	3041 in^3	4138 in^3
Self-weight of the section, SW	52.21 lb/ft^2	+25.00 lb/ft^2
Volume-to-surface ratio, V/S	1.4 in	-

Table 3.1. Section properties

ΣP_i = 135491 lb

y_{psp} = 13.4 in

4. Prestressing Force Loss Calculations

Since prestressing losses are highly sensitive to the location of prestressing strands, losses are calculated for each layer of strands seperately. Results given in Table 4.1 on the following page are the losses calculated for the strand layer that is closest to the extreme bottom fiber of the section. In elastic shortening loss calculation, elasticity modulus of precast concrete at transfer stage E_{pi} is calculated by assuming concrete has reached 70% of its strength at transfer stage ($f'_{cpi}=0.70f'_{cp}$)

4.1 Elastic shortening loss

$$\Delta f_{ps,el}=\frac{f_{cps}E_{ps}}{E_{pi}}$$

4.2 Steel relaxation at transfer stage

$$\Delta f_{ps,re1}=f'_{psi}\frac{logt_1}{10}\left[\frac{f'_{psi}}{f_{psy}}-0.55\right]$$

4.3 Creep

$$\Delta f_{ps,cr}=\frac{f_{cps,lt}\times 2\times E_{ps}}{E_p}$$

4.4 Shrinkage

Use V/S in inches and E_{ps} in psi; the resulting stress loss is in psi.

$$\Delta f_{ps,sh}=8.2\times 10^{-6}\times 1.0E_{ps}\left[1-0.06\frac{V}{S}\right][100-RH]$$

4.5 Steel relaxation at service stage

$$\Delta f_{ps,re1}=f'_{psi}\frac{logt_2}{10}\left[\frac{f'_{psi}}{f_{psy}}-0.55\right]$$

xL	f_{psi}	$-\Delta f_{ps,es}$	$-\Delta f_{ps,re1}$	$-\Delta f_{ps,cr}$	$-\Delta f_{ps,sh}$	$-\Delta f_{ps,re2}$
ft	$/f_{psu}$	$/f_{psu}$	$/f_{psu}$	$/f_{psu}$	$/f_{psu}$	$/f_{psu}$
0.00	0.750	0.682	0.658	0.545	0.522	0.505
1.80	0.750	0.685	0.661	0.557	0.534	0.514
3.60	0.750	0.688	0.664	0.568	0.545	0.521
5.40	0.750	0.691	0.666	0.578	0.555	0.527
7.20	0.750	0.693	0.668	0.587	0.564	0.532
9.00	0.750	0.695	0.670	0.594	0.571	0.537
10.80	0.750	0.697	0.671	0.600	0.577	0.540
12.60	0.750	0.698	0.672	0.605	0.582	0.543
14.40	0.750	0.699	0.673	0.608	0.585	0.545
16.20	0.750	0.700	0.673	0.610	0.587	0.546
18.00	0.750	0.700	0.673	0.611	0.588	0.547

Table 4.1. Prestressing losses

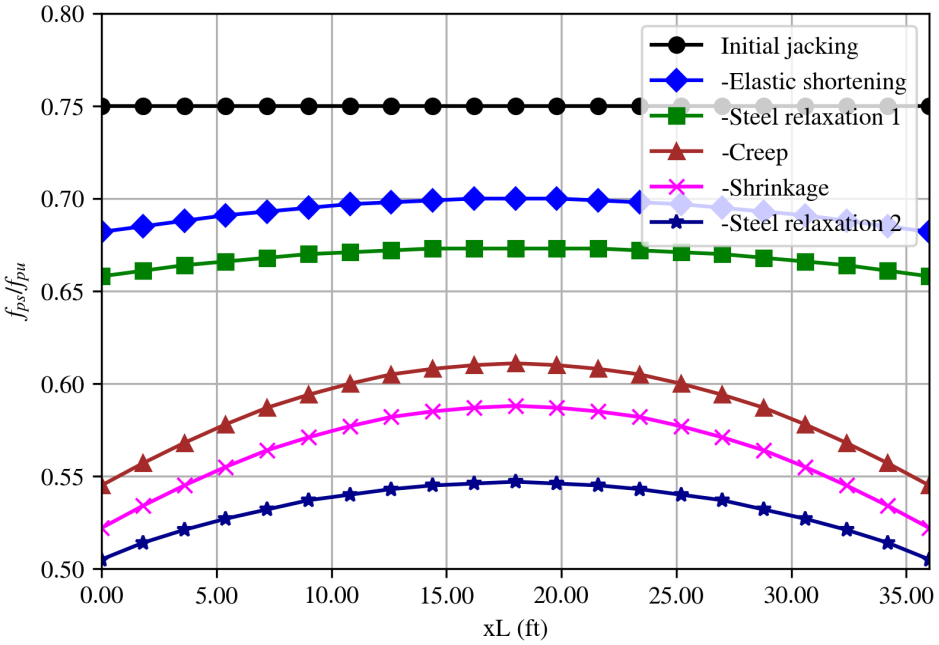


Figure 4.1. Prestressing losses

5. Strength Design

5.1 Bending Moment Strength Design

Bending moment strength is evaluated in two stages. Stage 1 is where the self-weight of the double TT and the topping weight acts on the members. The bending moment is resisted by the precast section. For each section, the ratio of prestressing strands necessary to resist Stage 1 loads is determined. At Stage 2, the remaining amount of strands resist the bending moments due to superimposed and live loads. At this stage, composite section resists the loads.

Nominal strength of the section is determined by using Equation 20.3.2.3.1 of ACI318-25 for prestressing strand stress:

$$f_{ps} = f_{psu}\left[1 - \frac{\gamma_{ps}}{\beta_1}\left[\rho_{ps}\frac{f_{psu}}{f'_c} + \frac{d_s}{d_{ps}}\frac{f_{sy}}{f'_c}(\rho_s - \rho'_s)\right]\right]$$

ACI318-25 Eq. 20.3.2.3.1

Since the member is fully prestressed, there is no contribution from mild steel longitudinal rebars to the bending moment strength (ρ_s=0, ρ'_s=0). γ_{ps} and β_1 is determined from ACI 318-25 Table 20.3.2.3.1 and Table 22.2.2.4.3, respectively. The nominal moment strength of the section is determined using equivalent compression stress block approach, as shown schematically in Figure 5.1.1 for Stage 1.

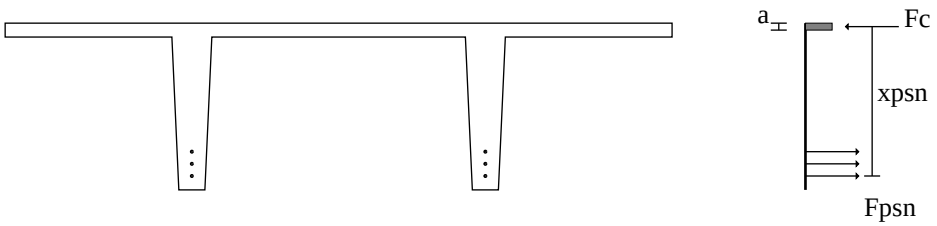


Figure 5.1.1. Schematic representation of equivalent compression stress block approach

The expressions to calculate the parameters in Figure 5.1.1 are given below. It should be noted that if equivalent compression stress block depth a falls below the flange thickness H1, the expression changes to take into account the web thickness. For brevity only the first case is shown below.

$$a = \frac{\Sigma F_{psn}}{\beta_1 B1 f_{cp}}$$

$$M_r = \Sigma F_{psn} x_{psn}$$

The overstrength factor ϕ is determined by checking if the section is within the region of undeveloped strand length as per ACI 318-25 21.2.3. The ductility requirement of ACI 318-25 9.6.2.1 is satisfied in the design by ensuring the ultimate bending moment strength is greater than $1.2M_{cr}$ at all sections, where M_{cr} is calculated using the expression below.

$$M_{cr} = \left[f_{rp} + \frac{\Sigma P_e}{A_p} + \frac{\Sigma P_e (y_{bp} - y_{psp})}{S_{bp}}\right] S_{bp}$$

xL	f_{ps}	ϕ	$\%_{ps1}$	M_{u1}	ϕM_{r1}
ft	ksi		%	lb-in	lb-in
0.00	266.56	0.75	5	0	185673
1.80	266.56	0.75	8	273771	278399
3.60	266.56	0.78	15	518723	578400
5.40	266.56	0.82	20	734858	810362
7.20	266.56	0.86	22	922175	956305
9.00	266.56	0.90	25	1080673	1110528
10.80	266.56	0.90	28	1210354	1221098
12.60	266.56	0.90	30	1311217	1331581
14.40	266.56	0.90	32	1383262	1441976
16.20	266.56	0.90	32	1426489	1441976
18.00	266.56	0.90	32	1440898	1441976

Table 5.1.1 Stage 1 bending moment strength analysis

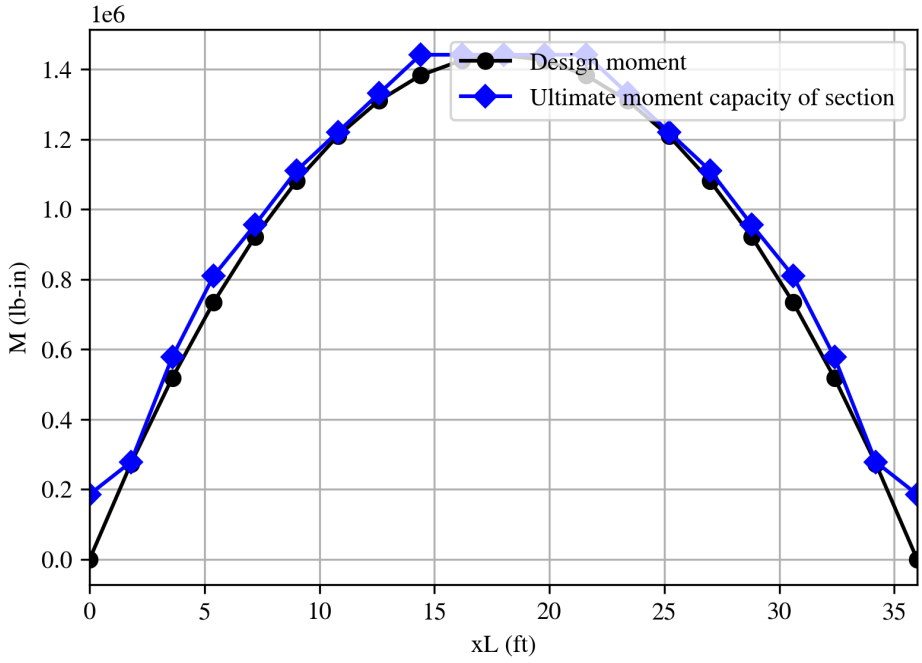


Figure 5.1.2 Bending moment strength vs. capacity / Precast section

xL	f_{ps}	ϕ	$\%_{ps2}$	M_{u2}	ϕM_{r2}	$1.2M_{cr2}$
ft	ksi		%	lb-in	lb-in	lb-in
0.00	265.09	0.75	95	0	3775963	916099
1.80	265.09	0.75	92	364989	3678693	916126
3.60	265.09	0.79	85	691559	3564210	916149
5.40	265.09	0.83	80	979708	3537108	916170
7.20	265.09	0.87	78	1229438	3599511	916187
9.00	265.09	0.90	75	1440748	3593556	916202
10.80	265.09	0.90	72	1613637	3475744	916213
12.60	265.09	0.90	70	1748107	3357796	916222
14.40	265.09	0.90	68	1844157	3239712	916229
16.20	265.09	0.90	68	1901787	3239712	916233
18.00	265.09	0.90	68	1920997	3239712	916234

Table 5.1.2. Stage 2 bending moment strength analysis

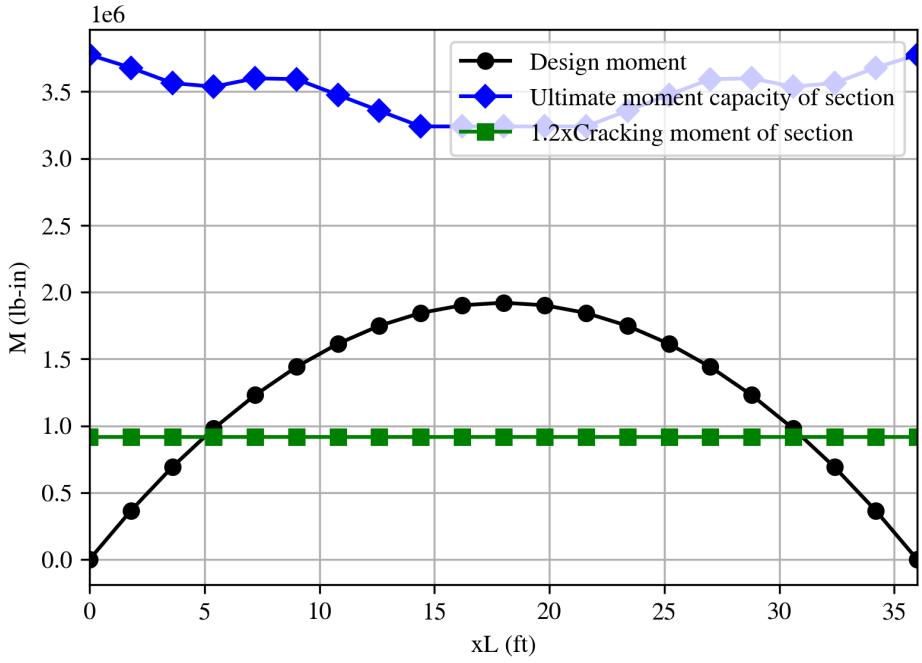


Figure 5.1.3. Bending moment strength vs. capacity / Composite section

5.2 Shear Strength Design

Shear strength design is performed in accordance with ACI 318-25 Section 22.5. The contribution to shear strength by concrete, V_c, is the lesser of the values calculated for flexure-shear strength V_{ci} and web-shear strength V_{cw}, as specified by ACI 318-25 22.5.6.

Flexure-shear strength V_{ci} is calculated as follows. Note that the expression becomes essentially infinite at the support, where M_{max} is equal to zero. It is expected since flexural cracking does not occur at zero moment (shear strength of concrete is governed by web-shear strength). Therefore, V_{ci} at support (x=0) is labeled as '-' in Table 5.2.1.

$$V_{ci} = max(0.6\sqrt{f_{cp}}B_{w1}d_{ps} + V_d + \frac{V_i M_{cre}}{M_{max}}, 1.7\sqrt{f_{cp}}B_{w1}d_{ps})$$

ACI 318-25 Eq. 22.5.6.2.1a

Web-shear strength V_{cw} is calculated as follows:

$$V_{cw} = (3.5\sqrt{f_{cp}} + 0.3f_{psc})B_{w1}d_{ps} + V_{ps}$$

ACI 318-25 Eq. 22.5.6.2.2

$$V_s >= \frac{V_u}{\varphi} - V_c$$

ACI 318-25 Eq. 22.5.8.1

The larger of required transferse reinforcement and minimum transferse reinforcement as required by ACI 318-25 Table 9.6.3.4 is provided. Also, sufficiency of the section dimensions are checked according to ACI 318-25 Eq. 22.5.1.2 using the expression below.

$$V_u <= \varphi\left[V_c + 8\sqrt{f_{cp}}B_{w1}d_{ps}\right]$$

ACI 318-25 Eq. 22.5.1.2

xL	ϕ	V_{ci}	V_{cw}	V_c	$\phi_{vs,prov}$	V_s	V_u	ϕV_r
ft		kips	kips	kips		kips	kips	kips
0.00	0.75	-	37.6	37.6	W2.9@6in	11.7	31.1	49.3
1.80	0.75	99.8	52.0	52.0	W2.9@6in	11.7	28.0	63.8
3.60	0.75	53.6	52.3	52.3	W2.9@6in	11.7	24.9	64.0
5.40	0.75	37.4	52.4	37.4	W2.9@6in	11.7	21.8	49.1
7.20	0.75	28.6	52.6	28.6	W2.9@6in	11.7	18.7	40.4
9.00	0.75	22.9	52.7	22.9	W2.9@6in	11.7	15.6	34.6
10.80	0.75	21.5	52.8	21.5	W2.9@6in	11.7	12.5	33.2
12.60	0.75	21.5	52.9	21.5	W2.9@6in	11.7	9.3	33.2
14.40	0.75	21.5	52.9	21.5	W2.9@6in	11.7	6.2	33.2
16.20	0.75	21.5	53.0	21.5	W2.9@6in	11.7	3.1	33.2
18.00	0.75	21.5	53.0	21.5	W2.9@6in	11.7	0.0	33.2

Table 5.2.1 Shear strength analysis

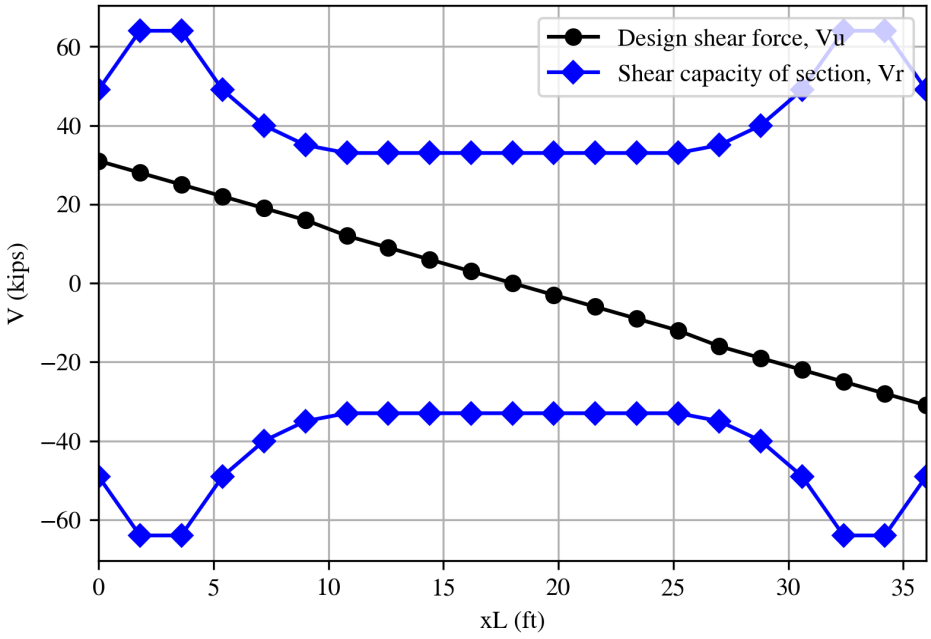


Figure 5.2.1. Shear strength analysis

6. Serviceability Checks

6.1 Stress Limit Checks

Stress limit checks are performed for three stages, which are described in Table 6.1.1 below.

Stage	Section	Loads	Losses
1. At transfer	Precast	Self weight Prestressing	Elastic shortening
2. Topping is cast, but not hardened	Precast	Topping	Steel relaxation 1
3. Service loads	Composite	Superimposed dead load Live load	Creep Shrinkage Steel relaxation 2

Table 6.1.1 Stress check stages

The stress limit checks for different stages are outlined below:

Stage	Stress limit check	Precaution
1. At transfer	$f_{cp1,min} > 0.7 f_{cip}$ (xL < H1+H2)	Increase f_{cip} till $0.80 f_{cp}$
	$f_{cp1,min} > 0.6 f_{cip}$ (xL > H1+H2)	Increase f_{cip} till $0.80 f_{cp}$
	$f_{cp1,max} < 6\sqrt{f_{cip}}$ (xL < H1+H2)	Provide bonded mild steel reinforcement to resist total tensile force
	$f_{cp1,max} < 3\sqrt{f_{cip}}$ (xL > H1+H2)	Provide bonded mild steel reinforcement to resist total tensile force
2. Topping is cast, but not hardened	$f_{cp2,max} < 12\sqrt{f_{cp}}$	Not allowed
3. Service loads	$f_{cp3,max} < 12\sqrt{f_{cp}}$	Not allowed
3a. Service loads (prestressing + sustained loads)	$f_{cp3,min} > 0.45 f_{cp}$	Not allowed
3b. Service loads (all loads)	$f_{cp3,min} > 0.60 f_{cp}$	Not allowed

Table 6.1.2 Stress limit checks

Stress-limit expressions use psi in this report.

The principles shown in Table 6.1.2 can be summarized as follows:

- The extreme fiber stress in tension in the precompressed tension zone calculated at service loads assuming an uncracked section shall not exceed $12\sqrt{f_{cp}}$. In other words, any section of the member shall not have cracked behaviour, described as Class C in ACI 318-25 Table 24.5.2.1. If this condition is not met with the prestressing strand quantity calculated in ultimate moment strength analysis, strand quantity will be increased until this condition is met.
- At Stage 1, extreme fiber stress in compression shall not exceed $0.70f_{cip}$ for sections that has distance less than H1+H2 from the ends of the member, and $0.60f_{cip}$ for all other sections. If this condition is not met, f_{cip} shall be increased up to $0.80f_{cp}$. If this is not enough to satisfy the condition as well, the section will be deemed unsufficient and the user will be recommended to increase f_{cp}. (ACI 318-25 24.5.3.1)
- At Stage 1, if the extreme fiber stress in tension exceeds $6\sqrt{f_{cip}}$ for sections that has distance less than H1+H2 from the ends of the member, and $3\sqrt{}_{cip}$ for all other sections, bonded mild steel reinforcement shall be provided to resist total tensile force. (ACI 318-25 24.5.3.2)
- For sustained loads (all loads excluding live loads), extreme fiber stress in compression shall not exceed $0.45f_{cp}$ or $0.45f_{ct}$ for all sections. Prestressing strands shall be increased until this condition is satisfied. (ACI 318-25 24.5.4.1)
- For all sustained loads including live loads, extreme fiber stress in compression shall not exceed $0.60f_{cp}$ or $0.60f_{ct}$ for all sections. Prestressing strands shall be increased until this condition is satisfied. (ACI 318-25 24.5.4.1)
- The two preceeding compressive stress checks shall be performed for top fibers of both precast concrete and topping concrete, since the latter typically has smaller compressive strength.

The expressions with which the stresses are calculated for different stages are shown below. Please note that **compressive stresses are negative, tensile stresses are positive.**

$$f_{cp1,bot} = -\frac{\sum P(t)}{A_p} - \frac{\sum P(t) y_{psp}}{S_{bp}} + \frac{M_{SW}}{S_{bp}}, \quad f_{cp1,top} = -\frac{\sum P(t)}{A_p} + \frac{\sum y_{psp}}{\frac{P(t)}{S_{tp}}} - \frac{M_{SW}}{S_{bp}}$$

$$f_{cp2,bot} = -\frac{\sum P(t)}{A_p} - \frac{\sum P(t) y_{psp}}{S_{bp}} + \frac{M_{SW} + M_{to}}{S_{bp}}, \quad f_{cp2,top} = -\frac{\sum P(t)}{A_p} + \frac{\sum P(t) y_{psp}}{S_{tp}} - \frac{M_{SW} + M_{to}}{S_{tp}}$$

$$f_{cp3,bot} = -\frac{\sum P(t)}{A_p} - \frac{\sum P(t) y_{psp}}{S_{bp}} + \frac{M_{SW} + M_{to}}{S_{bp}} + \frac{M_{SDL} + M_{LL}}{S_{bc}}, \quad f_{cp3,top} = -\frac{\sum P(t)}{A_p} + \frac{\sum P(t) y_{psp}}{S_{tp}} - \frac{M_{SW} + M_{to}}{S_{tp}} - \frac{M_{SDL} + M_{LL}}{S_{tc}}$$

$$f_{ct3,top} = -\frac{M_{SDL} + M_{LL}}{S_{tc}} \frac{E_t}{E_p}$$

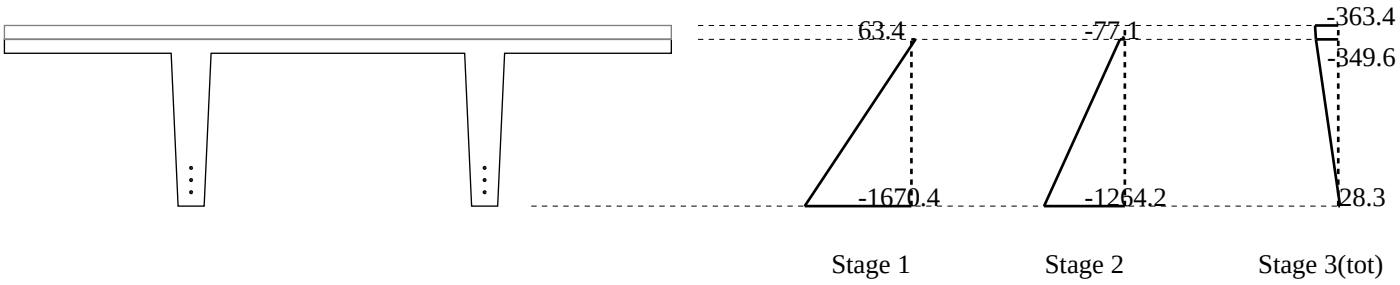


Figure 6.1.1 Stresses calculated for mid-span section (psi)

xL	Stage 1		Stage 2		Stage 3.a (sustained)			Stage 3.b (total)		
	$f_{cp1,bot}$	$f_{cp1,top}$	$f_{cp2,bot}$	$f_{cp2,top}$	$f_{cp3,bot}$	$f_{cp3,top}$	$f_{ct3,top}$	$f_{cp3,bot}$	$f_{cp3,top}$	$f_{ct3,top}$
ft	psi	psi	psi	psi	psi	psi	psi	psi	psi	psi
0.00	-2279.8	322.8	-2198.2	311.3	-1706.2	241.2	0.0	-1706.2	241.2	0.0
1.80	-2164.0	273.5	-2020.8	237.5	-1513.3	162.2	-32.6	-1377.7	129.1	-69.0
3.60	-2060.4	229.4	-1862.0	171.5	-1340.4	91.4	-61.7	-1083.3	28.8	-130.8
5.40	-1969.0	190.5	-1721.9	113.2	-1187.5	28.9	-87.4	-823.3	-59.8	-185.3
7.20	-1889.8	156.8	-1600.5	62.7	-1054.7	-25.4	-109.7	-597.7	-136.7	-232.6
9.00	-1822.8	128.2	-1497.7	20.0	-942.1	-71.3	-128.5	-406.6	-201.7	-272.5
10.80	-1767.9	104.9	-1413.7	-15.0	-849.9	-108.8	-143.9	-250.1	-254.9	-305.2
12.60	-1725.2	86.7	-1348.3	-42.1	-778.1	-138.1	-155.9	-128.3	-296.3	-330.7
14.40	-1694.8	73.7	-1301.6	-61.6	-726.8	-159.0	-164.5	-41.3	-325.9	-348.8
16.20	-1676.5	66.0	-1273.5	-73.2	-696.0	-171.5	-169.6	10.9	-343.7	-359.7
18.00	-1670.4	63.4	-1264.2	-77.1	-685.7	-175.7	-171.4	28.3	-349.6	-363.4

Table 6.1.3 Stresses at top and bottom fibers

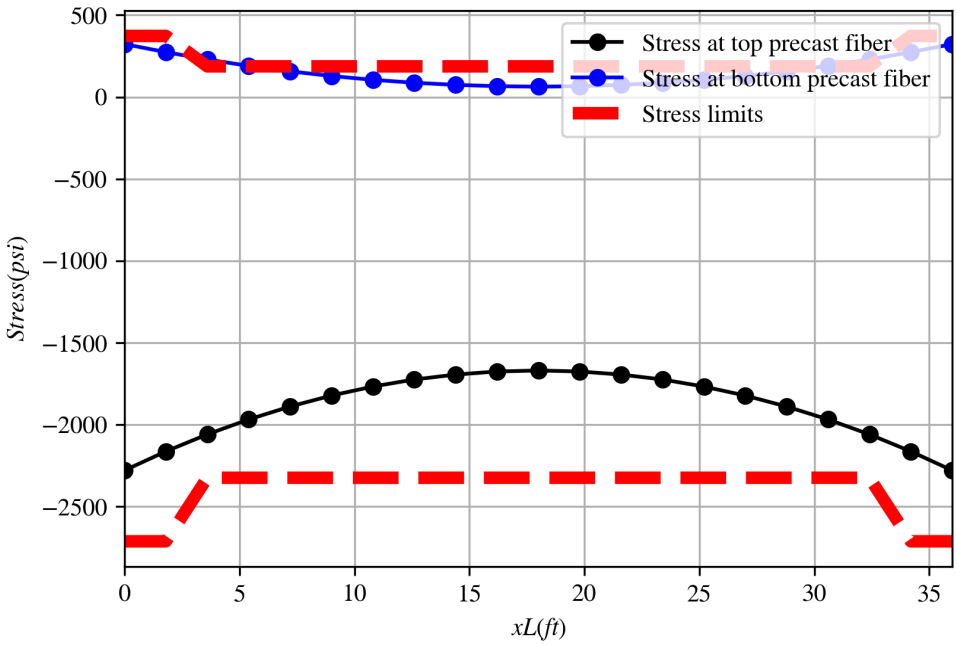
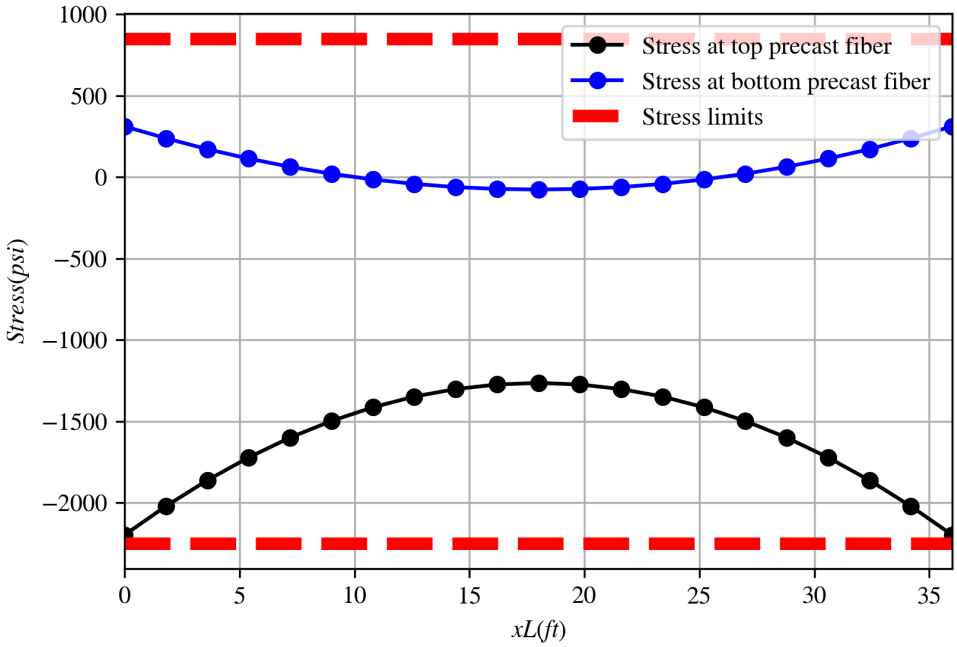


Figure 6.1.2 Stresses at top and bottom fibers at Stage 1

The tensile stress limits for transfer stage (Stage 1) given in ACI 318-25 24.5.3.2.1 are exceeded. For this reason, bonded mild steel reinforcement shall be provided at the flange to resist all tensile loads. The section at which demand is highest is at xL=3.60 ft.

$$A_{s,ft,req} = \frac{25729\text{lbs}}{30000\text{psi}} = 0.86\text{in}^2$$

The required mild reinforcement area is 0.86 in^2. 10x#3 rebars (1.10 in^2) are provided in the flange.



6.1.3 Stresses at top and bottom fibers at Stage 2

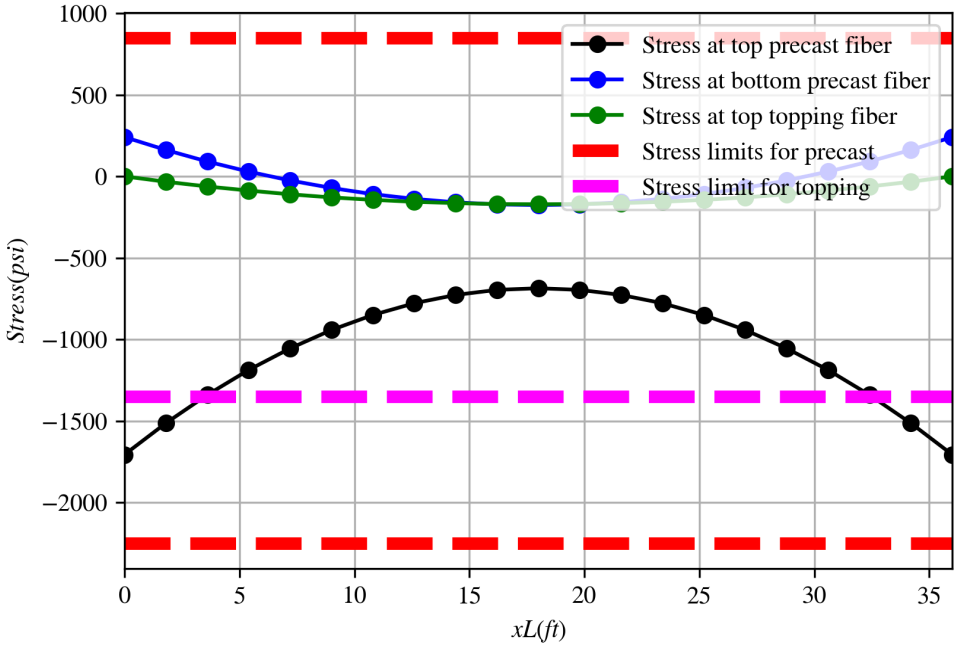


Figure 6.1.4 Stresses at top and bottom fibers at Stage 3a (sustained loads)

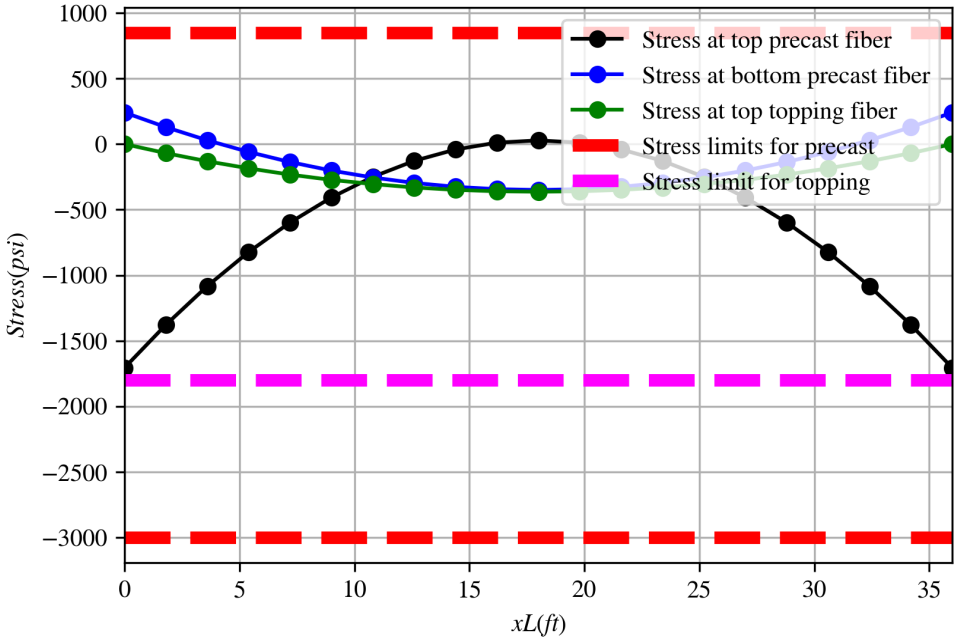


Figure 6.1.5 Stresses at top and bottom fibers at Stage 3b (total loads)

6.2 Deflections

Important note: Since deflection limits can be flexible, the design is not iteratively changed if the limits given in ACI 318-25 Table 24.2.2 are exceeded. However, in such a case, the deflections that exceed limits are written on red background in Table 6.2.3.

Deflections are calculated according to the principles specified in ACI 318-25 Section 24.2, taking into account time-dependent properties of the section and material at each stage. This approach is outlined in Table 6.2.1 below. The long-term modifiers are given in PCI Design Handbook 7th edition Table 5.8.2.

Deflection	Elasticity Modulus	Moment of Inertia	Long-term Modifier	
			At erection	Final
Prestressing	E_{pi}	I_p	1.80	2.20
Self weight	E_{pi}	I_p	1.85	2.40
Topping	E_p	I_p	-	2.30
Superimposed dead load	E_p	I_e	-	3.00
Live load	E_p	I_e	-	1.00

Table 6.2.1 Deflection analysis principles

As shown in Table 6.1.3, the extreme bottom-fiber stress under total service loads (28 psi) does not exceed the cracking threshold $7.5\sqrt{f_{cp}}$ (530 psi). Therefore, according to ACI 318-25 Table R24.5.2.1, gross sectional properties are used for deflection calculations.

The deflections calculated using the afore-mentioned principles are given in Table 6.2.2.

Deflection	δ_{tr}	PM_{er}	δ_{er}	δ_{se}	PM_{fi}	δ_{fi}
Prestressing	-0.64in	1.85	-1.19in	-0.64in	2.2	-1.41in
Self weight	0.21in	1.8	0.38in	0.21in	2.4	0.51in
Topping	-	-	-	0.09in	2.3	0.21in
Superimposed dead load	-	-	-	0.04in	3.0	0.12in
Live load	-	-	-	0.18in	1.0	0.18in
Total	**-0.43in**	**-**	**-0.81in**	**-0.12in**	**1.0**	**-0.39in**
	L/-1005	**-**	**L/-533**	**L/-3600**	**1.0**	**L/-1108**

Table 6.2.2 Deflections

	Deflection	Deflection limitation
Immediate deflection due to LL	L/2400	L/360
Total deflection after attachment of nonstructural elements	L/1440	L/480

Table 6.2.3 Deflection limits given in ACI 318-25 Table 24.2.2

7. Miscelleneous calculations

7.1 Dapped-end design

The dapped-end design is done according to ACI/PCI 319-25 Section 16.6. Notation for design parameters are shown in Figure 7.1.1. Since the Double-Tee is fully prestressed and the web thickness is less than 5.0 in, two-legged stirrup reinforcement is not practical in this layout. Therefore, one-legged reinforcement is provided at the dapped end.

Results for dapped end reinforcement design are shown in Table 7.1.1 below. Design details are given in the following parts of this section.

Reinforcement	Color in drawing	Required	Provided
$A_{s,dap}$		$0.25in^2$	$2x\#4(0.39in^2)$
$A_{sh,dap}$		$0.35in^2$	$1x\#6(0.44in^2)$
$A_{h,dap}$		$0.32in^2$	$1x\#4(0.39in^2)$
$A_{v,dap}$		$0.10in^2$	$3x\#3(0.33in^2)$

Table 7.1.1 Dapped-end reinforcement

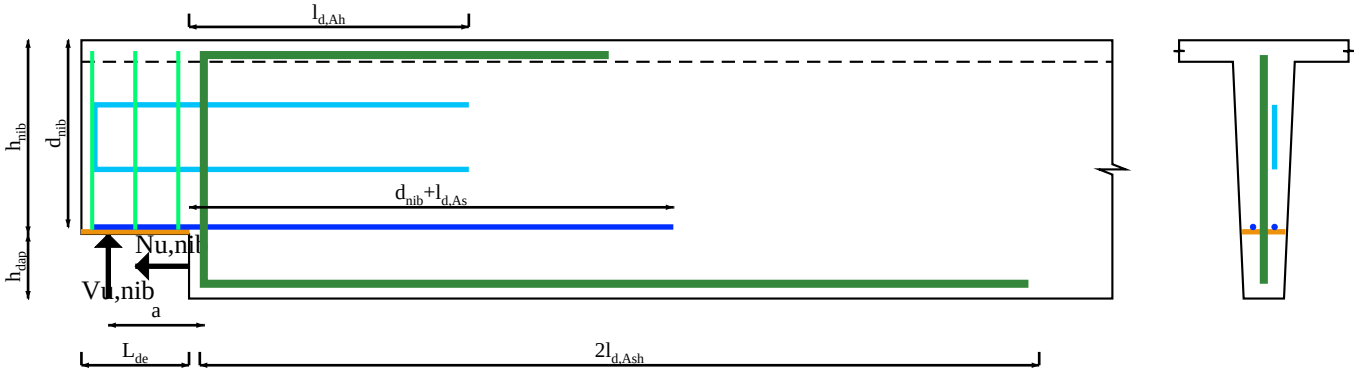


Figure 7.1.1 Dapped-end design parameters

$$a = 8.9\,\text{in}$$

$$V_{u,nib} = 15.6\,\text{kips}$$

$$N_{u,nib} = 0.2V_{u,nib} = 3.1\,\text{kips}$$

The sufficiency of nib dimension for dapped-end is checked in terms of nib shear strength $V_{n,nib}$ according to ACI/PCI 319-25 Section 16.6.2.4.

$$V_{n,nib} = 5 \times \sqrt{5000}\,\text{psi} \times 3.75\,\text{in} \times 17.35\,\text{in} = 23.0\,\text{kips} > V_{u,nib}/0.75 = 20.8\,\text{kips}$$

The required reinforcement given in Figure 7.1.1 and Table 7.1.1 are calculated as follows, according to PCI Design Handbook 7th edition Section 5.6.3.

$$A_{s,nib,1} = \frac{1}{\phi f_y}\left[V_u\frac{a}{d} + N_u\frac{h}{d}\right] = \frac{1}{0.75\times 60000}\left[15564.3\frac{8.88}{17.35} + 3112.9\frac{18.00}{17.35}\right] = 0.25\,\text{in}^2$$

$$A_{s,nib,2} = \frac{2V_u}{3\phi f_y \mu_e} + \frac{N_u}{\phi f_y} = \frac{2\times 15564.3}{3\times 0.75\times 60000\times 3.40} + \frac{3112.9}{0.75\times 60000} = 0.14\,\text{in}^2$$

$$A_{s,nib} = \max(A_{s,nib,1}, A_{s,nib,2}) = 0.25\,\text{in}^2$$

$$A_{h,nib} = \max(0.5(A_s - A_n),\ 0.005B_{w1}d_{nib}) = \max(0.5(0.14 - 0.07), 0.005\times 3.75\times 17.35) = 0.32\,\text{in}^2$$

$$A_{sh,nib} = \frac{V_u}{\phi f_y} = \frac{15564.3}{0.75\times 60000} = 0.35\,\text{in}^2$$

$$A_{v,nib} = \frac{1}{2f_y}\left[V_u - 2.00bd\sqrt{f'_c}\right] = \frac{1}{2\times 60000}\left[15564.3 - 2.00\times 3.75\times 17.35\times\sqrt{5000}\right] = 0.10\,\text{in}^2$$

The development lengths of the rebars given in Figure 7.1.1 are calculated according to ACI 318-25 Section 25.4.2 (Table 25.4.2.3).

$$l_{d,As} = \frac{3}{50}\times\frac{60000}{71}\times 0.50 = 26.00\,\text{in}$$

$$l_{d,Ah} = \frac{3}{50}\times\frac{60000}{71}\times 0.50 = 26.00\,\text{in}$$

$$l_{d,Ash} = \frac{3}{50}\times\frac{60000}{71}\times 0.75 = 39.00\,\text{in}$$

7.2 Flange design

The transverse reinforcement required at flange is determined by analysing the transverse moment acting on it. Composite action is not considered in the design. In other words, only the flange carry the entire loads without any contribution from topping concrete. Figure 7.2.1 shows the moment diagram.

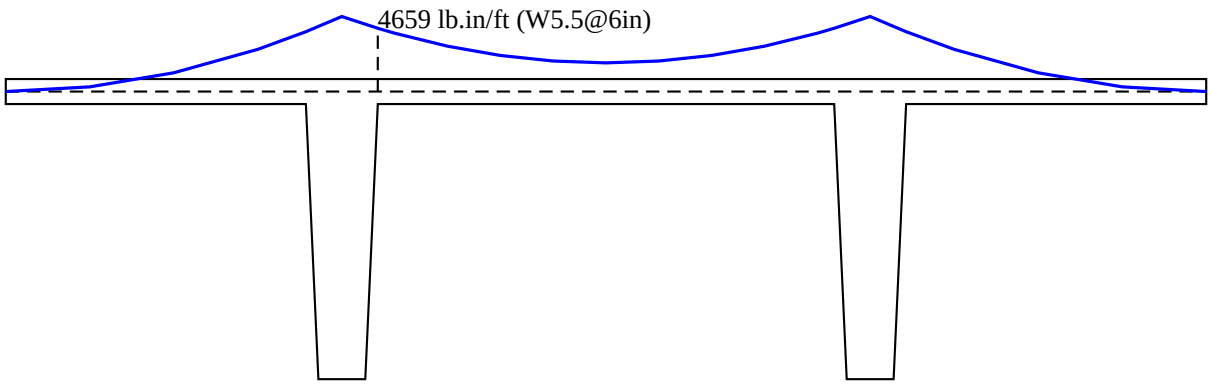


Figure 7.2.1 Transverse flexural moment acting on the flange

7.3 Composite behaviour design

The composite action of the member is ensured by providing horizontal shear transfer at the interface according to ACI 318-25 Section 16.4 and PCI Design Handbook 7th ed. Section 5.3.5.

$$l_v = \frac{L}{2} = \frac{432}{2}\text{in} = 216\text{in}$$

$$F_{h1} = 0.85 f'_{ct} B_1 H_t = 0.85(3000)(96.00)(2.00) = 489.6\text{kips}$$

$$F_{h2} = A_{ps} f_{psu} = (0.92)(270.0) = 247.9\text{kips}$$

$$F_h = max(F_{h1}, F_{h2}) = 489.6\text{kips}$$

According to ACI 318-25 Table 16.4.4.1, since $F_h < 80 B1 l_v$ (489.6 kip < 1658.9 kip), no horizontal shear reinforcement is required by this check. Horizontal shear force transfer will be provided by "cementitious bond realized by concrete placed against hardened concrete that is clean, free of laitance, and at least tightly textured."

8. References

[1] ACI Committee 318, Building Code for Structural Concrete—Code Requirements and Commentary (ACI CODE-318-25). Farmington Hills, MI, USA: American Concrete Institute, 2025.

[2] Precast/Prestressed Concrete Institute, PCI Design Handbook: Precast and Prestressed Concrete, 7th ed., MNL-120-10. Chicago, IL, USA: Precast/Prestressed Concrete Institute, 2010.

[3] ACI Committee 319, Structural Precast Concrete - Code Requirements and Commentary (ACI/PCI CODE-319-25). Farmington Hills, MI, USA: American Concrete Institute, 2025.